Optimum Impedance and Dimensions for Strip Transmission Line*

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Summary—This paper makes use of the higher mode limitations on the dimensions of symmetrical strip transmission line to derive the permissible dimensions at any given frequency and characteristic impedance. In conjunction with Cohn's results for the attenuation in strip transmission line these are used to obtain the maximum achievable Q at any frequency and the optimum characteristic impedance; that is, the impedance providing the lowest attenuation. This provides the basis for selecting the characteristic impedance for resonant elements in strip line filters and other applications wherein the lowest possible attenuation is desired. Conclusions are also reached regarding the best form factor (ratio of strip thickness to ground-plane spacing) for a given characteristic impedance.

Introduction

IN MANY microwave applications it is desirable to use a section of transmission line having the lowest possible attenuation. This is particularly true in the case of narrow band microwave filters where lengths of transmission line are used as resonant elements. In such an application, the characteristic impedance and the line dimensions may usually be chosen arbitrarily. It is necessary, therefore, to know how the attenuation varies with these parameters and what limitations are imposed upon them. Although this information is well known for coaxial line and uniconductor waveguides, it is not generally known for strip transmission lines. It is the purpose of this paper to present this information for symmetrical strip line comprising a flat strip center conductor centrally located between, and parallel to, two parallel ground planes.

As has been shown by Cohn,[1] the attenuation of symmetrical strip line decreases as the characteristic impedance is decreased for a constant ground-plane spacing, and decreases as the ground-plane spacing is increased for a constant characteristic impedance. Therefore, strip transmission line does not have an optimum impedance for fixed outer conductor size analogous to the case of coaxial line. However, if the outer conductor size of coaxial line is not limited, the optimum impedance is limited by the first circumferential mode; it is 92.6 ohms, and produces the absolute minimum attenuation.[2] Similarly, by considering the size limitations due to higher modes in symmetrical strip lines, we may deduce the optimum impedance for these lines.

Higher Mode Limitations on Line Dimensions

One limit on the line dimensions is imposed by the TM modes, the lowest of which has a cut-off wavelength equal to twice the ground-plane spacing. This gives an absolute upper limit to the ground-plane spacing. For spacings less than this, a possible circumferential TE mode can also impose a limit. The cut-off frequency of this TE mode depends on the strip width and the ground-plane spacing. The TE cut-off wavelength may be calculated[3] from the analogous E-plane bifurcation in rectangular waveguide, and for the lowest mode is given by

$$\lambda_c = 2w + 4d \tag{1}$$

where d is the distance from the edge of the strip to the open-circuit point, and is given for infinitesimally thin strips by,[4]

$$d = \frac{D}{\pi}\ln 2 + \frac{\lambda}{2\pi}\left[S_1\left(\frac{2D}{\lambda}\right) - 2S_1\left(\frac{D}{\lambda}\right)\right] \tag{2}$$

with

$$S_1(x) = \sum_{n=1}^{\infty}\left(\arcsin\frac{x}{n} - \frac{x}{n}\right).$$

(See Marcuvitz[4] for tabulation of arcsine sum functions.)

The meaning of w and D are as shown in Fig. 1, where the field configuration for this mode is shown and Fig. 1(b) shows the "uniform field" equivalent of Fig. 1(a); that is, a line having the same propagation constant and characteristic impedance, but no fringing capacitance. Although (2) holds only for an infinitesimally thin strip, the results may be put in a form which takes account of the strip thickness, t. Consider the "uniform field" equivalent for a thick strip as shown in Fig. 2(a). For the line in Fig. 2(a) to have the same characteristic impedance and cut-off wavelength as that in Fig. 1(b), we must have,

$$w' = w + 2d, \qquad D' = D + t.$$

Now the line in Fig. 2(a) is the equivalent of some actual line shown in Fig. 2(b). Therefore, a line with a strip of thickness, t, has the same cut-off wavelength as

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[1] S. Cohn, "Problems in strip transmission lines," IRE Trans., vol. MTT-3, pp. 119–126; March, 1955.

[2] G. L. Ragan, "Microwave Transmission Circuits," McGraw-Hill Book Co., Inc., New York, N. Y., p. 146; 1948.

[3] A. A. Oliner, "Theoretical Developments in Symmetrical Strip Transmission Line," presented at Symposium on Modern Advances in Microwave Techniques, Polytechnic Inst. of Brooklyn, Brooklyn, N. Y.; November, 1954.

[4] N. Marcuvitz, "Waveguide Handbook," McGraw-Hill Book Co., Inc., New York, N. Y., p. 353; 1951.

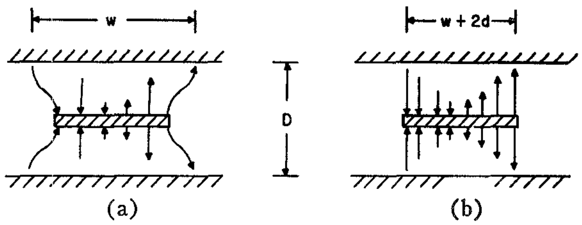

Fig. 1—Zero-thickness strip line. (a) Actual line; (b) uniform field equivalent.

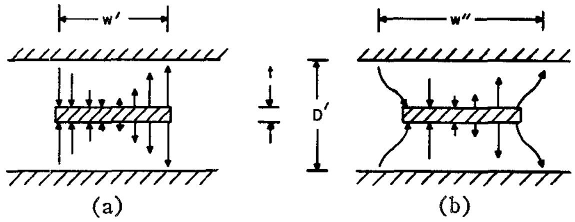

Fig. 2—Thick-strip line. (a) Uniform field equivalent; (b) actual line.

a line of the same characteristic impedance but with a zero-thickness strip and a ground-plane spacing less than that of the former by the amount, t.

We may combine (1) and (2) and write

$$\frac{w}{D} = \frac{v}{2}\frac{1}{Df_c} - \frac{2\ln 2}{\pi} - \frac{v}{\pi}\frac{1}{Df_c}\left[S_1\left(\frac{2}{v}Df_c\right) - 2S_1\left(\frac{1}{v}Df_c\right)\right] \quad (3)$$

where v is the phase velocity and f_c is the cut-off frequency. Eq. (3) gives the maximum value of w/D at the cut-off frequency from which the minimum permissible characteristic impedance may be calculated.[5,6] If now this characteristic impedance is plotted as a function of $\sqrt{\epsilon}Df_c$, the curves of Fig. 3 result, where those for thick strips are obtained by multiplying the abscissas of the $t=0$ curve by $D'/(D'-t)$. Therefore, as used in Fig. 3, D refers to any line regardless of strip thickness. These curves are useful in determining the operating frequency limit for a line of given dimensions. We will make further use of them, however, in deriving the optimum characteristic impedance.

It should be noted that the assumption made in using an analysis based on the waveguide E-plane bifurcation and the "uniform field" equivalents is that there is no higher mode interaction between the two edges of the strip. This assumption is true providing that the strip is not too narrow. For the ranges of characteristic impedance and t/D used in Fig. 3, the value of w/D is not less than 0.35 for values of $\sqrt{\epsilon}Df_c$ up to 5.75. Furthermore, this minimum value of w/D holds for thin strips, whereas the minimum value of w/D is even larger for thicker strips. It can be expected, therefore, that for practical strip line geometries higher mode interaction will not be a serious factor and the results based on this assumption will be valid.

[5] S. Cohn, "Characteristic impedance of the shielded-strip transmission line," IRE Trans., vol. MTT-2, pp. 52–57; July, 1954.

[6] R. H. T. Bates, "The characteristic impedance of the shielded slab line," IRE Trans., vol. MTT-4, pp. 28–33; January, 1956.

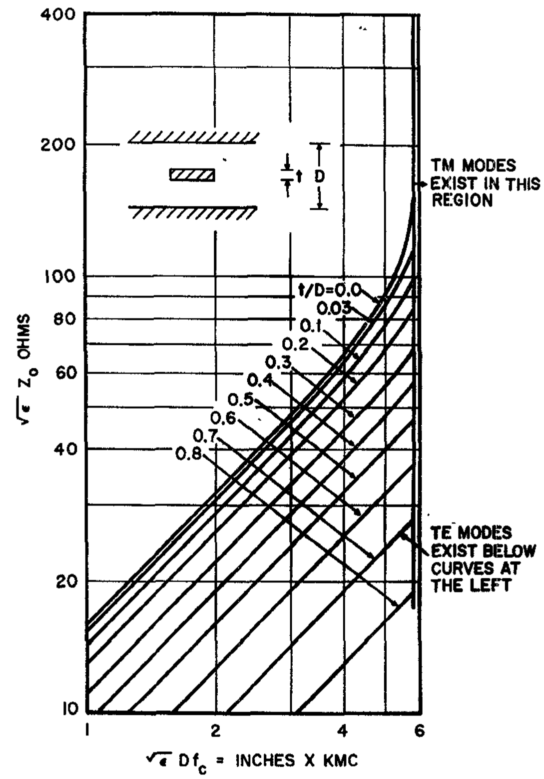

Fig. 3—Minimum characteristic impedance vs higher mode cut-off frequency.

Optimum Characteristic Impedance

From Cohn's calculations of losses in symmetrical strip line[1] the ground-plane spacing should be as great as possible for low loss, but from the curves in Fig. 3 it is seen that a high impedance must then be used which in itself implies high loss. Due to the nonlinear relations involved, however, it appears that there should be a minimum loss. That is, over the range of characteristic impedance to be considered, inspection of the curves used by Cohn[1] show that the unloaded Q of a resonant line as a function of the frequency, ground-plane spacing and impedance can be approximated by a linear function,

$$Q\sqrt{f} = Df(A - B\sqrt{\epsilon}Z_0)$$

where $A \gg B$ and both are constants. The lower asymptote of each of the curves in Fig. 3 is

$$\sqrt{\epsilon}Z_0 = 15.95\sqrt{\epsilon}(D-t)f$$

so that in this region the maximum Q is given by

$$Q\sqrt{f} = Df[A - C(D-t)f]$$

where C is a constant and the second term on the right is much smaller than the first over the range of Df to be considered. Therefore, $Q\sqrt{f}$ is an increasing function of Df in this range. As the curve in Fig. 3 departs from this asymptote, however, the minimum impedance rapidly approaches a very large value and therefore $Q\sqrt{f}$ must decrease as Df increases. Due to the unwieldy functions involved, this maximum is most easily found by graphical methods. The results of these calculations for copper conductors are shown in Fig. 4, for several values of t/D from which it is seen that the minimum loss for very thin strips is obtained for a characteristic impedance of about 95 ohms at a ground-plane spacing of $t+0.44\lambda/\sqrt{\epsilon}$ inches. This has been verified experimentally for thin strips and is true for values of t/D up to 0.14 at which point the limit for the TM mode is reached. For t/D greater than 0.14 the maximum achievable Q is limited only by the ground-plane spacing, the optimum impedance being indicated by the intersection of the parametric curves in Fig. 3 with the line $\sqrt{\epsilon}Df_c=5.9$, which is the cut-off condition for the lowest TM mode.

The curves in Fig. 4 also show that there is an optimum set of dimensions for minimum loss. That is, the ratio $t/D=0.25$ and a characteristic impedance of 76 ohms, with a ground-plane spacing of one-half wavelength at the operating frequency, produces the absolute minimum attenuation. The maximum obtainable resonator Q for this case is $2.25\times10^4/\sqrt{f}$ which is comparable to the $2.1\times10^4/\sqrt{f}$ obtainable for coaxial line using copper conductors. It should also be mentioned that, although the approximations used in calculating the attenuation as a function of characteristic impedance do not permit the full range of values of t/D to be included, Cohn's calculations have been extended by the writer to higher values of t/D. Although the limits imposed do not permit a positive statement, it does appear that the minimum attenuation for a *fixed-ground-plane spacing* will also be obtained for $t/D=0.25$ over the impedance range 80 to 130 ohms. The improvement over the case $t/D=0.1$ is not great, however, and for all practical purposes the latter may be used for fixed ground-plane spacing throughout the range of impedances most used.

The accuracy of these calculations is limited by the accuracy of Cohn's formulas for attenuation. These are admittedly approximate, but are accurate to ±4 per cent which is certainly sufficient for most all applications.

Practical Applications of Results

It should be pointed out that the optimum impedance and ground-plane spacing arrived at by this procedure dictate operation at the cut-off frequency of one or more higher modes. In actual engineering practice it would be necessary to provide some margin of safety by operating below these cut-off frequencies, particularly in the case of the TM modes which will radiate.[3] In some applications it may be permissible to operate under conditions where the TE modes can exist, and, in fact, what might be called super-Q resonators have been made at this Laboratory under these conditions.

To provide sufficient reactive attenuation of the higher modes of the strip line, an analysis can be made

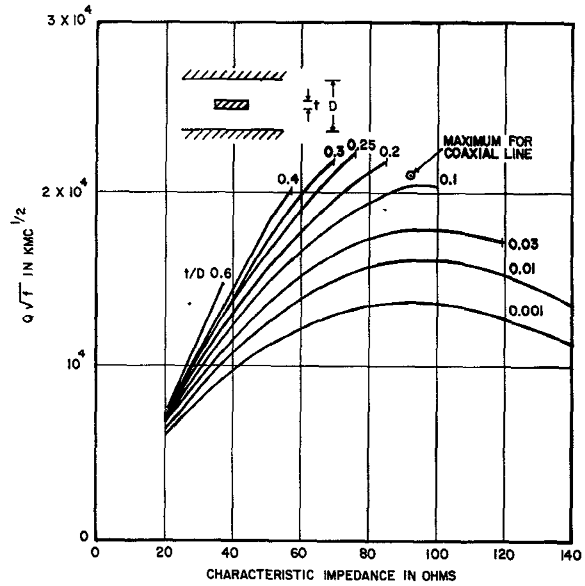

Fig. 4—Maximum obtainable Q vs characteristic impedance.

using the expression for the attenuating effect of a waveguide below cut-off,

$$L = 54.5\frac{d}{\lambda_c}\sqrt{1-(\lambda_c/\lambda)^2}\text{ db per length, } d.$$

This may be put in a form more useful for the present purpose giving the following equation for the new operating conditions in terms of the desired higher mode attenuation,

$$Df = Df_c\sqrt{1-(L/27.3)^2},$$

where L is the desired attenuation in db per ground-plane spacing. L has a maximum value of 27.3, as may be seen from the equation for L by letting $\lambda\rightarrow\infty$ and $d=D$, since D/λ_c has a maximum of $\frac{1}{2}$. If the same reactive attenuation is required for both TE and TM modes, then the optimum characteristic impedance will remain unchanged since the whole operating curve in Fig. 3 is shifted to the left. If the attenuation is only required for the TM modes, then the optimum characteristic impedance will be given by the intersection of the parametric curves in Fig. 3 with the new constant Df line. The maximum Q may be found by transferring these limiting values of impedance to the curves in Fig. 4. For arbitrary amounts of reactive attenuation for the two types of higher mode the user can readily find the optimum characteristic impedance from the principles used above. Particular care in these matters must be taken in designing filters as the unwanted modes can seriously affect the coupling between the filter elements.

Conclusion

To summarize the results obtained, several points may be brought out. The first of these is that caution must be exercised in regard to TE modes in low impedance circuits. For example, it is not generally appreciated that, for typical values such as $D=\lambda/4$ and $t/D=0.1$, the TE mode cut-off corresponds to a characteristic impedance of 43 ohms. Whereas the existence of this mode does not necessarily cause serious trouble, it may often explain discrepancies between experimental results and those calculated on the basis of a pure TEM mode. As is obvious from Fig. 3, this may be avoided by using a thicker strip.

The second point is the existence of an optimum characteristic impedance for obtaining the lowest attenuation. The value of this optimum will depend on the desired higher mode attenuation. Because the conditions will vary widely for different applications, the data presented cover the case of operation at the cut-off frequency of both the lowest TE and TM modes. This condition produces the lowest possible loss. For practical applications, however, the method for obtaining the optimum in other cases has been outlined.

A few final words should be said in regard to an interesting point shown by the curves in Fig. 3 and 4. It is assumed that one usually wishes to operate with the lowest possible line losses and this generally imples a high value of D, and therefore, of Df. As to the strip dimensions for lowest line loss an examination of the curves in Figs. 3 and 4 shows that for low values of characteristic impedance, it is desirable (see Fig. 4) and often necessary (see Fig. 3) to use high values of t/D. From the curves for characteristic impedance given by Bates[6] it is seen that high values of t/D imply small values of w/D. On the other hand, Bates also shows that a high characteristic impedance can *only* be obtained with small values of t/D. Therefore, it may be concluded that, in addition to the preceding considerations of the optimum characteristic impedance, one may make the generalization that a high impedance line with lowest loss should be in the familiar strip line from ($t \ll w$) whereas low impedance lines with lowest loss should be made with much thicker strips, in some cases with the strip thickness exceeding the strip width ($t>w$).

Deflection of Waveguide Subjected to Internal Pressure*

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Summary—**The pressure carrying capacity of a large range of standard waveguide sizes can be readily determined by the use of formulas presented in this paper. The derivation of these formulas is obtained by a continuous beam analogy and comparable test results are shown which substantiate the validity of the theoretical analysis.**

Where high pressure conditions prevent the use of standard waveguide, these same formulas are utilized in the development of special high-strength lightweight guide. Techniques for designing such waveguide, including the use of a honeycomb sandwich construction, are discussed.

THE DEVELOPMENT of radar systems of increasing range has been brought about largely by the use of greater and greater power. In order to increase the power handling capacity of the microwave packages, pressurization is utilized to prevent electrical breakdown. It is, therefore, extremely desirable to be able to determine quickly the pressure carrying capacity of a given waveguide and, when standard waveguide cannot safely carry the required pressure, to be able to design special guide of minimum weight and/or cost.

The derivation of formulas that express the relationship of wall thickness to pressurization capacity is presented for a considerable range of waveguide sizes. The criteria for this relationship are 1) that the waveguide should not permanently distort, and 2) that the elastic deflection should not exceed the amount permissible for satisfactory microwave use. The problem is approached both analytically and empirically with good correlation between the two methods.

Fig. 1(a) depicts a typical cross section of unpressurized guide. The application of internal pressure results in distortion as shown in Fig. 1(b). The question frequently arises, "How can the short wall bend inward when the pressure should be forcing it outward?" A simplified explanation of this phenomenon is that the corner moment, due to the relatively greater length of the long wall, is sufficient to more than overcome the internal pressure on the short wall, resulting in an inward deflection. This is borne out by both the derived formulas and actual test results.

The pressurized waveguide cross section is considered to be similar to a uniformly loaded continuous beam of an infinite number of spans (or a simple beam with end moments[1]) as shown in Fig. 2. The analysis that follows pertains to unsupported waveguide which, as a practical consideration, means that it is applicable to sections that

* Manuscript received by the PGMTT, March 13, 1957; revised manuscript received, May 13, 1957.

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[1] T. N. Anderson, "Rectangular and ridge waveguide," IRE Trans., vol. MTT-4, pp. 201–209; October, 1956.